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Deformed Supersymmetry in Non(anti)commutative $\mathcal{N}=2$ Supersymmetric $U(1)$ Gauge Theory

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Abstract

We study $\mathcal{N}=2$ supersymmetric $U(1)$ gauge theory in non(anti)commutative $\mathcal{N}=2$ harmonic superspace with the chirality preserving non-singlet deformation parameter. By solving the Wess-Zumino gauge preserving conditions for the analytic superfield, we construct the deformed $\mathcal{N}=(1,0)$ supersymmetry transformation for component fields up to the first order in the deformation parameter.

Supersymmetric field theories in deformed superspace[1] have been recently attracted much interest, partly motivated by studying superstring effective field theories on the D-branes with graviphoton background [2, 3, 4]. Non(anti)commutative superspace is a deformed superspace with nonanticommutative Grassmann coordinates. Field theories in non(anti)commutative $\mathcal{N} = 1$ superspace ($\mathcal{N} = 1/2$ superspace) is defined by the fermionic version of the Moyal $*$-product. The deformed action can be constructed in terms of superfields, whose procedure is the same as field theories in noncommutative spacetime. Compare to noncommutative field theories, the action usually contains a finite number of deformed terms. The effects of non(anti)commutativity can be calculated explicitly. There are a lot of works on field theories in deformed $\mathcal{N} = 1$ superspace from both perturbative and non-perturbative points of view[5, 6, 7, 8].

It is interesting to study the deformation of extended superspace [9]–[17] because there is a variety of choices for the deformation. In the case of the deformed $\mathcal{N} = 2$ harmonic superspace, the deformation parameter can be decomposed into the singlet deformation part and the non-singlet part with respect to R-symmetry group $SU(2)_R$ [11, 12]. In contrast to the case of $\mathcal{N} = 1/2$ superspace, the deformed action takes in general the form of infinite power series in the deformation parameters, which is similar to noncommutative field theories. In particular, $\mathcal{N} = 2$ supersymmetric gauge theory with the singlet deformation of $\mathcal{N} = 2$ harmonic superspace has been recently studied in [16] and [17], where a field redefinition analogous to the Seiberg-Witten map [18] in theories with space-space noncommutativity was found (see also [19, 14]). The component action is fully determined in [17].

In the case of the non-singlet deformation parameterized by C, we have studied the $\mathcal{N} = 2$ supersymmetric $U(1)$ gauge theory in the non(anti)commutative $\mathcal{N} = 2$ harmonic superspace by using component formalism[13]. Choosing the Wess-Zumino (WZ) gauge for the analytic superfield, we have written down the deformed action up to the first order in the deformation parameter. We have shown that the commutative gauge transformation does not preserve the WZ gauge due to the $*$-product and one need to perform additional C-dependent gauge transformation in order to recover the WZ gauge. We have also made a field redefinition such that the component fields transform canonically under the gauge transformation.

In this letter, we will study the chiral supersymmetry transformation ($\mathcal{N} = (1, 0)$ in the sense of [11]) of the $\mathcal{N} = 2$ supersymmetric $U(1)$ gauge theory in the chirality preserving non(anti)commutative $\mathcal{N} = 2$ harmonic superspace. Since two of the present authors discussed the exact gauge and supersymmetry in the singlet deformation case [16], we will study the non-singlet deformation of the superspace. We will determine the deformed supersymmetry up to the order $O(C)$ under which the $O(C)$ action in [13] is invariant. The field redefinition given in [13], which makes the deformed gauge transformation to be the same as the one in the ordinary abelian theory, is applied to the supersymmetry transformation. We will find that the component transformation laws are simplified by the redefinition.

We begin with reviewing the non(anti)commutative $\mathcal{N} = 2$ supersymmetric $U(1)$ gauge theory based on [13]. Let $(x^\mu, \theta^i_\alpha, \bar{\theta}^i_{\dot\alpha})$ be the coordinates of $\mathcal{N} = 2$ (rigid) superspace. Here $\mu = 0, 1, 2, 3$ are indices of spacetime with Euclidean signature. $\alpha, \dot\alpha = 1, 2$ denote the spinor indices and $i = 1, 2$ labels the doublet of the $SU(2)_R$ R-symmetry. We use the antisymmetric tensor $\varepsilon^{\alpha\beta}$ with $\varepsilon^{12} = -\varepsilon_{12} = 1$ for raising and lowering spinor indices as in [20] but for R-symmetry indices we use ϵ^{ij} with $\epsilon^{12} = -\epsilon_{21} = -1$. In the Euclidean spacetime, θ^i_α and $\bar{\theta}^i_{\dot\alpha}$ are independent spinors. The supersymmetry generators Q^i_α, $\bar{Q}_{\dot\alpha i}$ and the supercovariant derivatives D^i_α, $\bar{D}_{\dot\alpha i}$ are defined by

$$
\begin{aligned}
Q^i_\alpha &= \frac{\partial}{\partial\theta^\alpha_i} - i(\sigma^\mu)_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha i}\frac{\partial}{\partial x^\mu}, \quad \bar{Q}_{\dot\alpha i} = -\frac{\partial}{\partial\bar{\theta}^{\dot\alpha i}} + i\theta^\alpha_i(\sigma^\mu)_{\alpha\dot\alpha}\frac{\partial}{\partial x^\mu}, \\
D^i_\alpha &= \frac{\partial}{\partial\theta^\alpha_i} + i(\sigma^\mu)_{\alpha\dot\alpha}\bar{\theta}^{\dot\alpha i}\frac{\partial}{\partial x^\mu}, \quad \bar{D}_{\dot\alpha i} = -\frac{\partial}{\partial\bar{\theta}^{\dot\alpha i}} - i\theta^\alpha_i(\sigma^\mu)_{\alpha\dot\alpha}\frac{\partial}{\partial x^\mu}.
\end{aligned} \tag{1}
$$

The $\mathcal{N} = 2$ harmonic superspace [21] is introduced by adding the harmonic variables $u^\pm_i$ to the $\mathcal{N} = 2$ superspace coordinates. The variables $u^\pm_i$ form an $SU(2)$ matrix and satisfy the conditions $u^{+i}u^-_i = 1$ and $\overline{u^{+i}} = u^-_i$. The completeness condition for $u^\pm_i$ is given by $u^+_i u^-_j - u^+_j u^-_i = \epsilon_{ij}$. Using $u^\pm_i$, the $SU(2)_R$ indices can be projected into two parts with ± 1 $U(1)(\subset SU(2)_R)$ charges. For example, we define the supercovariant derivatives $D^\pm_\alpha$ and $\bar{D}^\pm_{\dot\alpha}$ by $D^\pm_\alpha = u^\pm_i D^i_\alpha$, $\bar{D}^\pm_{\dot\alpha} = u^\pm_i \bar{D}^i_{\dot\alpha}$. D^i_α is solved by $D^\pm_\alpha$ such as $D^\pm_\alpha = u^+_i D^-_\alpha - u^-_i D^+_\alpha$ with the help of the completeness condition. In the harmonic superspace formalism, an important ingredient is an analytic superfield rather than the $\mathcal{N} = 2$ chiral superfield. An analytic superfield $\Phi(x, \theta, \bar\theta, u)$ is defined by $D^+_\alpha\Phi = \bar{D}^+_{\dot\alpha}\Phi = 0$. It is convenient to write

this analytic superfield in terms of analytic basis: $x^\mu_A = x^\mu - i(\theta^i\sigma^\mu\bar\theta^j + \theta^j\sigma^\mu\bar\theta^i)u^+_i u^-_j$, $\theta^\pm_\alpha = u^\pm_i\theta^i_\alpha$ and $\bar\theta^\pm_{\dot\alpha} = u^\pm_i\bar\theta^i_{\dot\alpha}$. In the analytic basis, an analytic superfield Φ is functions of $(x^\mu_A, \theta^+, \bar\theta^+, u)$: $\Phi = \Phi(x^\mu_A, \theta^+, \bar\theta^+, u)$. We now introduce the nonanticommutativity in the $\mathcal{N} = 2$ harmonic superspace by using the $*$-product:

$$f * g(\theta) = f(\theta)\exp(P)g(\theta), \quad P = -\frac{1}{2}\overleftarrow{Q^i_\alpha} C^{\alpha\beta}_{ij}\overrightarrow{Q^j_\beta}, \tag{2}$$

where $C^{\alpha\beta}_{ij}$ is some constants. With this $*$-product, we have following (anti)commutation relations:

$$\{\theta^\alpha_i, \theta^\beta_j\}_* = C^{\alpha\beta}_{ij}, \quad [x^\mu_L, x^\nu_L]_* = [x^\mu_L, \theta^\alpha_i]_* = [x^\mu_L, \bar\theta^{\dot\alpha i}]_* = 0, \quad \{\bar\theta^{\dot\alpha i}, \bar\theta^{\dot\beta j}\}_* = \{\bar\theta^{\dot\alpha i}, \theta^\alpha_j\}_* = 0, \tag{3}$$

where $x^\mu_L \equiv x^\mu + i\theta_i\sigma^\mu\bar\theta^i$. The deformation parameter $C^{\alpha\beta}_{ij}$ is symmetric under the exchange of pairs of indices (αi),(βj): $C^{\alpha\beta}_{ij} = C^{\beta\alpha}_{ji}$. We decompose the nonanticommutative parameter $C^{\alpha\beta}_{ij}$ into the symmetric and antisymmetric parts with respect to the $SU(2)$ indices, such as

$$C^{\alpha\beta}_{ij} = C^{\alpha\beta}_{(ij)} + \frac{1}{4}\epsilon_{ij}\varepsilon^{\alpha\beta}C_s. \tag{4}$$

Here we denote $A_{(i_1\cdots i_n)}$ by the symmetrized sum of $A_{i_1\cdots i_n}$ over indices $i_1, \cdots, i_n$. $C^{\alpha\beta}_{ij}$ with zero $C^{\alpha\beta}_{(ij)}$ corresponds to the singlet deformation [11, 12]. For superfields A and B, the $*$-product takes the form

$$A * B = AB + APB + \frac{1}{2}AP^2B + \frac{1}{6}AP^3B + \frac{1}{24}AP^4B, \quad P^5 = 0. \tag{5}$$

Since P commutes with the supercovariant derivatives D, the chiral structure is preserved by this deformation. In the analytic basis, one can compute the $*$-product by using $Q^i_\alpha = u^{+i}Q^-_\alpha - u^{-i}Q^+_\alpha$. For example we have

$$\{\theta^\eta, \theta^{\eta'}\}_* = C^{\eta\eta'\alpha\beta}, \quad [x^\mu_A, x^\nu_A]_* = 4C^{--\mu\nu}(\bar\theta^+)^2, \quad [x^\mu_A, \theta^\eta_\alpha]_* = -2iC^{-\eta\beta\alpha}(\sigma^\mu\bar\theta^+)_\beta, \tag{6}$$

where $\eta, \eta' = \pm$, $C^{\eta\eta'\mu\nu} = u^{\eta i}u^{\eta' j}C^{\mu\nu}_{ij}$, $C^{\mu\nu}_{ij} \equiv C^{\alpha\beta}_{ij}\sigma^{\mu\nu\gamma}_\alpha\varepsilon_{\beta\gamma}$ and $\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu)$. Since we will consider the non-singlet deformation, we put $C_s = 0$ in the following.

We now construct the action of $\mathcal{N} = 2$ supersymmetric $U(1)$ gauge theory in this non(anti)commutative superspace. We introduce an analytic superfield $V^{++}(\zeta, u)$ with $\zeta = (x^\mu_A, \theta^+, \bar\theta^+)$ by covariantizing the harmonic derivative $D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} - 2i\theta^+\sigma^\mu\bar\theta^+\frac{\partial}{\partial x^\mu_A} +$

$\theta^{+\alpha}\frac{\partial}{\partial\theta^{-\alpha}}+\bar{\theta}^{+\dot{\alpha}}\frac{\partial}{\partial\theta^{-\dot{\alpha}}} \to \nabla^{++}=D^{++}+iV^{++}$. Generalizing the construction in [22, 23], the action is given by

$$S_*=\frac{1}{2}\sum_{n=2}^{\infty}\int d^4xd^8\theta du_1\dots du_n\frac{(-i)^n}{n}\frac{V^{++}(1)*\cdots*V^{++}(n)}{(u_1^+u_2^+)\cdots(u_n^+u_1^+)}. \tag{7}$$

where $V^{++}(i)=V^{++}(\zeta_i,u_i)$, $\zeta_i=(x_A,\theta_i^+,\bar{\theta}_i^+)$ and $d^8\theta=d^4\theta^+d^4\theta^-$ with $d^4\theta^\pm=d^2\theta^\pm d^2\bar{\theta}^\pm$. The harmonic integral is defined by the rules: (i) $\int du f(u)=0$ for $f(u)$ with non-zero $U(1)$ charge. (ii) $\int du 1=1$. (iii) $\int du u^+_{(i_1}\cdots u^+_{i_n}u^-_{j_1}\cdots u^-_{j_n)}=0,\ (n\geq 1)$. The action (7) is invariant under the gauge transformation

$$\delta^*_\Lambda V^{++}=-D^{++}\Lambda+i[\Lambda,V^{++}]_*, \tag{8}$$

with an analytic superfield Λ. The generic superfield $V^{++}(\zeta,u)$ includes infinitely many auxiliary fields. Most of these fields are gauged away except the lowest component fields in the harmonic expansion. One can take the WZ gauge

$$\begin{aligned} V^{++}_{WZ}(x_A,\theta^+,\bar{\theta}^+,u) &= -i\sqrt{2}(\theta^+)^2\bar{\phi}(x_A)+i\sqrt{2}(\bar{\theta}^+)^2\phi(x_A)-2i\theta^+\sigma^\mu\bar{\theta}^+A_\mu(x_A) \\ &\quad +4(\bar{\theta}^+)^2\theta^+\psi^i(x_A)u_i^- -4(\theta^+)^2\bar{\theta}^+\bar{\psi}^i(x_A)u_i^- \\ &\quad +3(\theta^+)^2(\bar{\theta}^+)^2D^{ij}(x_A)u_i^-u_j^-, \end{aligned} \tag{9}$$

which is convenient to study the theory in the component formalism.

The component action S_* in the WZ gauge can be expanded in a power series of the deformation parameter C. In [13], we have computed the $O(C)$ action explicitly. The quadratic part $S_{*,2}$ in S_* is the same as the commutative one:

$$S_{*,2}=\int d^4x\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}+\phi\partial^2\bar{\phi}-i\psi^i\sigma^\mu\partial_\mu\bar{\psi}_i+\frac{1}{4}D^{ij}D_{ij}\right\}. \tag{10}$$

The cubic part $S_{*,3}$ in S_* is of order $O(C)$ and given by

$$\begin{aligned} S_{*,3} &= \int d^4x\left[-\frac{2\sqrt{2}}{3}iC^{\alpha\beta}_{(ij)}\psi^i_\alpha(\sigma^\nu\partial_\nu\bar{\psi}^j)_\beta\bar{\phi}-2\sqrt{2}iC^{\alpha\beta}_{(ij)}\psi^i_\alpha(\sigma^\nu\bar{\psi}^j)_\beta\partial_\nu\bar{\phi}\right. \\ &\quad +\frac{2}{3}iC^{\alpha\beta}_{(ij)}A_\mu(\sigma^\mu\bar{\psi}^i)_\alpha(\sigma^\nu\partial_\nu\bar{\psi}^j)_\beta-iC^{\mu\nu}_{(ij)}\bar{\psi}^i\bar{\psi}^jF_{\mu\nu} \\ &\quad \left.+\sqrt{2}C^{\mu\nu}_{(ij)}D^{ij}A_\mu\partial_\nu\bar{\phi}+\frac{1}{\sqrt{2}}C^{\mu\nu}_{(ij)}D^{ij}F_{\mu\nu}\bar{\phi}\right]. \end{aligned} \tag{11}$$

Note that here we have already dropped the C_s dependent terms. We will refer

$$S_{*,2} + S_{*,3} \tag{12}$$

as the $O(C)$ action.

In the commutative case, the gauge parameter $\Lambda = \lambda(x_A)$ preserves the WZ gauge and gives rise to the gauge transformation for component fields. In the non(anti)commutative case, however, the gauge transformation (8) with the same gauge parameter does not preserve the WZ gauge because of the C-dependent terms arising from the commutator. In order to preserve the WZ gauge, one must include the C-dependent terms. The gauge parameter is shown to take the form

$$\begin{aligned} \lambda_C(\zeta, u) &= \lambda(x_A) + \theta^+\sigma^\mu\bar{\theta}^+\lambda_\mu^{(-2)}(x_A, u; C) + (\bar{\theta}^+)^2\lambda^{(-2)}(x_A, u; C) \\ &\quad + (\bar{\theta}^+)^2\theta^{+\alpha}\lambda_\alpha^{(-3)}(x_A, u; C) + (\theta^+)^2(\bar{\theta}^+)^2\lambda^{(-4)}(x_A, u; C), \end{aligned} \tag{13}$$

which has been determined by solving the WZ gauge preserving conditions expanded in harmonic modes [13]. The gauge transformation is also fully determined, which reads

$$\begin{aligned} \delta^*_{\lambda_C} A_\mu &= -\partial_\mu\lambda + O(C^2), \\ \delta^*_{\lambda_C}\phi &= O(C^2), \\ \delta^*_{\lambda_C}\psi_{\alpha i} &= \frac{2}{3}(\varepsilon C_{(ij)}\sigma^\mu\bar{\psi}^j)_\alpha\,\partial_\mu\lambda + O(C^2), \\ \delta^*_{\lambda_C} D_{ij} &= 2\sqrt{2}C^{\mu\nu}_{(ij)}\partial_\mu\lambda\partial_\nu\bar{\phi} + O(C^2) \\ \delta^*_{\lambda_C}(\text{others}) &= 0. \end{aligned} \tag{14}$$

The $O(C)$ action is invariant under the $O(C)$ gauge transformation (14).

These gauge transformations are not canonical. But if we redefine the component fields such as

$$\begin{aligned} \hat{A}_\mu &= A_\mu + O(C^2), \\ \hat{\phi} &= \phi + O(C^2), \quad \hat{\bar{\phi}} = \bar{\phi}, \\ \hat{\psi}_{\alpha i} &= \psi_{\alpha i} + \frac{2}{3}(\varepsilon C_{(ij)}\sigma^\mu\bar{\psi}^j)_\alpha A_\mu + O(C^2), \quad \hat{\bar{\psi}}^{\dot{\alpha}} = \bar{\psi}^{\dot{\alpha}} \\ \hat{D}_{ij} &= D_{ij} + 2\sqrt{2}C^{\mu\nu}_{(ij)}A_\mu\partial_\nu\bar{\phi} + O(C^2), \end{aligned} \tag{15}$$

the newly defined fields are shown to transform canonically: $\delta^*_{\lambda_C}\hat{A}_\mu = -\partial_\mu\lambda$, $\delta^*_{\lambda_C}(\text{others}) = 0$. In terms of redefined fields, the $O(C)$ action can be written as

$$
\begin{aligned}
S_{*,2} + S_{*,3} &= \int d^4x \Big[-\frac{1}{4}\hat{F}_{\mu\nu}(\hat{F}^{\mu\nu} + \tilde{\hat{F}}^{\mu\nu}) + \hat{\phi}\partial^2\hat{\bar{\phi}} - i\hat{\psi}^i\sigma^\mu\partial_\mu\hat{\bar{\psi}}_i + \frac{1}{4}\hat{D}^{ij}\hat{D}_{ij} \\
&\quad - 2\sqrt{2}iC^{\alpha\beta}_{(ij)}\hat{\psi}^i_\alpha(\sigma^\mu\hat{\bar{\psi}}^j)_\beta\partial_\mu\hat{\bar{\phi}} - \frac{2\sqrt{2}}{3}iC^{\alpha\beta}_{(ij)}\hat{\psi}^i_\alpha(\sigma^\mu\partial_\mu\hat{\bar{\psi}}^j)_\beta\hat{\bar{\phi}} \\
&\quad - iC^{\mu\nu}_{(ij)}\hat{\bar{\psi}}^i\hat{\bar{\psi}}^j\hat{F}_{\mu\nu} + \frac{1}{\sqrt{2}}C^{\mu\nu}_{(ij)}\hat{D}^{ij}\hat{F}_{\mu\nu}\hat{\bar{\phi}} + O(C^2)\Big], \qquad (16)
\end{aligned}
$$

where $\hat{F}_{\mu\nu} = \partial_\mu\hat{A}_\nu - \partial_\nu\hat{A}_\mu$.

Now we study the supersymmetry transformation that is generated by the chiral part of the supersymmetry generators: the $\mathcal{N} = (1,0)$ supersymmetry generated by Q^i_α. The deformed supersymmetry transformation of the gauge multiplet,

$$
\begin{aligned}
\delta^*_\xi V^{++}_{WZ} &= -i\sqrt{2}(\theta^+)^2\delta^*_\xi\bar{\phi}(x_A) + i\sqrt{2}(\bar{\theta}^+)^2\delta^*_\xi\phi(x_A) - 2i\theta^+\sigma^\mu\bar{\theta}^+\delta^*_\xi A_\mu(x_A) \\
&\quad + 4(\bar{\theta}^+)^2\theta^+\delta^*_\xi\psi^i(x_A)u^-_i - 4(\theta^+)^2\bar{\theta}^+\delta^*_\xi\bar{\psi}^i(x_A)u^-_i \\
&\quad + 3(\theta^+)^2(\bar{\theta}^+)^2\delta^*_\xi D^{ij}(x_A)u^-_i u^-_j, \qquad (17)
\end{aligned}
$$

is given by

$$
\delta^*_\xi V^{++}_{WZ} = \tilde{\delta}_\xi V^{++}_{WZ} + \delta^*_\Lambda V^{++}_{WZ}, \qquad (18)
$$

where

$$
\tilde{\delta}_\xi V^{++}_{WZ} = \left(-\xi^{+\alpha}Q^-_\alpha + \xi^{-\alpha}Q^+_\alpha\right)V^{++}_{WZ} \qquad (19)
$$

and $\delta^*_\Lambda V^{++}_{WZ}$ is a deformed gauge transformation of V^{++}_{WZ} with an appropriate analytic gauge parameter $\Lambda(\zeta, u)$ to retain the WZ gauge:

$$
\delta^*_\Lambda V^{++}_{WZ}(\zeta, u) = -D^{++}\Lambda(\zeta, u) + i[\Lambda, V^{++}_{WZ}]_*(\zeta, u). \qquad (20)
$$

We will denote the analytic gauge parameter as

$$
\begin{aligned}
\Lambda(\zeta, u) &= \lambda^{(0,0)}(x_A, u) + \bar{\theta}^+_{\dot{\alpha}}\lambda^{(0,1)\dot{\alpha}}(x_A, u) + \theta^{+\alpha}\lambda^{(1,0)}_\alpha(x_A, u) + (\bar{\theta}^+)^2\lambda^{(0,2)}(x_A, u) \\
&\quad + (\theta^+)^2\lambda^{(2,0)}(x_A, u) + \theta^+\sigma^\mu\bar{\theta}^+\lambda^{(1,1)}_\mu(x_A, u) + (\bar{\theta}^+)^2\theta^{+\alpha}\lambda^{(1,2)}_\alpha(x_A, u) \\
&\quad + (\theta^+)^2\bar{\theta}^+_{\dot{\alpha}}\lambda^{(2,1)\dot{\alpha}}(x_A, u) + (\theta^+)^2(\bar{\theta}^+)^2\lambda^{(2,2)}(x_A, u), \qquad (21)
\end{aligned}
$$

where $\lambda^{(n,m)}(x_A, u)$ is the $(\theta^+)^n(\bar{\theta}^+)^m$-component.

From eq. (18), the equations to determine the deformed supersymmetry transformation laws are obtained as follows:

$$
\begin{aligned}
0 &= 2i(\xi^+\sigma^\mu)_{\dot\beta}\varepsilon^{\dot\beta\dot\alpha}A_\mu - \partial^{++}\lambda^{(0,1)\dot\alpha} - 2\lambda^{(1,0)\alpha}(\varepsilon C^{++}\sigma^\mu)_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}A_\mu, && (22)\\
0 &= -2\sqrt{2}i\xi^+_\alpha\bar\phi - \partial^{++}\lambda^{(1,0)}_\alpha - 2\sqrt{2}(\varepsilon C^{++}\lambda^{(1,0)})_\alpha\bar\phi, && (23)\\
\sqrt{2}i\delta^*_\xi\phi &= 4\xi^+\psi^i u^-_i + 4i\lambda^{(1,0)\alpha}(\varepsilon C^{++}\psi^i)_\alpha u^-_i \\
&\quad - \partial^{++}\lambda^{(0,2)} - C^{++\alpha\beta}(\sigma^\nu\bar\sigma^\mu\varepsilon)_{\alpha\beta}\lambda^{(1,1)}_\mu A_\nu, && (24)\\
-\sqrt{2}i\delta^*_\xi\bar\phi &= 0, && (25)\\
-2i\delta^*_\xi A_\mu &= 4\xi^+\sigma_\mu\bar\psi^i u^-_i + 4i\lambda^{(1,0)\alpha}(\varepsilon C^{++}\sigma^\mu\bar\psi^i)_\alpha u^-_i \\
&\quad - \partial^{++}\lambda^{(1,1)}_\mu - \sqrt{2}C^{++\alpha\beta}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\lambda^{(1,1)}_\nu\bar\phi, && (26)\\
4\delta^*_\xi\psi^i_\alpha u^-_i &= -2(\sigma^\mu\bar\sigma^\nu\xi^-)_\alpha\partial_\nu A_\mu + 6\xi^+_\alpha D^{ij}u^-_i u^-_j \\
&\quad - i(\sigma^\nu\partial_\nu\lambda^{(0,1)})_\alpha - 2\sqrt{2}i(\varepsilon C^{+-}\sigma^\nu\partial_\nu\lambda^{(0,1)})_\alpha\bar\phi \\
&\quad - 6i(\varepsilon C^{++}\lambda^{(1,0)})_\alpha D^{ij}u^-_i u^-_j + 2i\lambda^{(1,0)\beta}(\varepsilon C^{+-}\sigma^\nu\bar\sigma^\mu\varepsilon)_{\beta\alpha}\partial_\nu A_\mu \\
&\quad + 2i\partial_\nu\lambda^{(1,0)}_\alpha C^{+-\beta\gamma}(\sigma^\mu\bar\sigma^\nu\varepsilon)_{\beta\gamma}A_\mu - 2i(\sigma^\mu\bar\psi^i)_\alpha u^-_i C^{++\gamma\delta}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\gamma\delta}\lambda^{(1,1)}_\nu \\
&\quad - \partial^{++}\lambda^{(1,2)}_\alpha - 2\sqrt{2}(\varepsilon C^{++}\lambda^{(1,2)})_\alpha\bar\phi + (\varepsilon C^{++}\sigma^\mu\lambda^{(2,1)})_\alpha A_\mu, && (27)\\
-4\delta^*_\xi\bar\psi^{\dot\alpha i}u^-_i &= 2\sqrt{2}(\xi^-\sigma^\mu)_{\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\partial_\mu\bar\phi + i\partial_\mu\lambda^{(1,0)\alpha}\sigma^\mu_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha} \\
&\quad + 2\sqrt{2}i\partial_\nu\left\{\lambda^{(1,0)\alpha}(\varepsilon C^{+-}\sigma^\nu)_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\bar\phi\right\} - \partial^{++}\lambda^{(2,1)\dot\alpha}, && (28)\\
3\delta^*_\xi D^{ij}u^-_i u^-_j &= -4i\xi^-\sigma^\mu\partial_\mu\bar\psi^i u^-_i + 4\partial_\nu\left\{\lambda^{(1,0)\alpha}(\varepsilon C^{+-}\sigma^\nu\bar\psi^i)_\alpha u^-_i\right\} \\
&\quad - i\partial^\mu\lambda^{(1,1)}_\mu - \sqrt{2}iC^{+-\alpha\beta}(\sigma^\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\partial_\nu(\lambda^{(1,1)}_\mu\bar\phi) - \partial^{++}\lambda^{(2,2)}. && (29)
\end{aligned}
$$

Inserting the harmonic expansions of gauge parameters into the above equations, one obtains a set of recursive relations for harmonic modes, which can be solved order by order in C. Up to the $O(C)$ terms, the associated gauge parameter Λ is given by the following components:

$$
\begin{aligned}
\lambda^{(1,0)\alpha} &= -i2\sqrt{2}\xi^{-\alpha}\bar\phi + i4(\xi_m\varepsilon C_{(kl)})^\alpha\bar\phi^2\left(u^{+(k}u^{-l}u^{-m)} - \frac{8}{3}\epsilon^{k(l}u^{-m)}\right) + O(C^2),\\
\lambda^{(0,1)}_{\dot\alpha} &= -i2(\xi^-\sigma^\mu)_{\dot\alpha}A_\mu \\
&\quad + i2\sqrt{2}(\xi_m\varepsilon C_{(kl)}\sigma^\mu)_{\dot\alpha}\bar\phi A_\mu\left(u^{+(k}u^{-l}u^{-m)} - \frac{8}{3}\epsilon^{k(l}u^{-m)}\right) + O(C^2),\\
\lambda^{(1,1)}_\mu &= 2(\xi^-\sigma_\mu\bar\psi^-) \\
&\quad + 4\sqrt{2}(\xi_m\varepsilon C_{(kl)}\sigma_\mu\bar\psi_n)\bar\phi\left(u^{+(k}u^{-l}u^{-m}u^{-n)} - \frac{9}{4}\epsilon^{k(l}u^{-m}u^{-n)}\right) + O(C^2),
\end{aligned}
$$

$$
\begin{aligned}
\lambda^{(0,2)} &= 2(\xi^-\psi^-) + \frac{8\sqrt{2}}{3}(\xi_m\varepsilon C_{(kl)}\psi_n)\bar{\phi}\left(u^{+(k}u^{-l}u^{-m}u^{-n)} - \frac{9}{4}\epsilon^{k(l}u^{-m}u^{-n)}\right) \\
&\quad + \frac{4}{3}(\xi_m\varepsilon C_{(kl)}\sigma^\mu\bar{\psi}_n)A_\mu\left(u^{+(k}u^{-l}u^{-m}u^{-n)} - \frac{9}{4}\epsilon^{k(l}u^{-m}u^{-n)}\right) + O(C^2), \\
\lambda^{(1,2)\alpha} &= 2\xi^{-\alpha}D^{--}, \\
&\quad - 4\sqrt{2}(\xi_p\varepsilon C_{(kl)})^\alpha D_{mn}\bar{\phi}\left(u^{+(k}u^{-l}u^{-m}u^{-n}u^{-p)} - \frac{32}{15}\epsilon^{k(l}u^{-m}u^{-n}u^{-p)}\right) \\
&\quad + i4(\xi_p\varepsilon C_{(kl)})^\alpha(\bar{\psi}_m\bar{\psi}_n)\left(u^{+(k}u^{-l}u^{-m}u^{-n}u^{-p)} - \frac{32}{15}\epsilon^{k(l}u^{-m}u^{-n}u^{-p)}\right) \\
&\quad + 4\sqrt{2}(\xi_m\varepsilon C_{(kl)}\sigma^{\mu\nu})^\alpha\partial_\mu(A_\nu\bar{\phi})u^{-(k}u^{-l}u^{-m)} \\
&\quad - \frac{8\sqrt{2}}{3}(\xi_m\sigma^{\mu\nu}\varepsilon C_{(kl)})^\alpha(\partial_\mu A_\nu\bar{\phi} - A_\nu\partial_\mu\bar{\phi})u^{-(k}u^{-l}u^{-m)} \\
&\quad + \frac{2\sqrt{2}}{3}(\xi_m\varepsilon C_{(kl)})^\alpha A_\mu\partial^\mu\bar{\phi}u^{-(k}u^{-l}u^{-m)} \\
&\quad - 2\sqrt{2}(\xi_m\varepsilon C_{(kl)})^\alpha\partial^\mu A_\mu\bar{\phi}u^{-(k}u^{-l}u^{-m)} + O(C^2), \\
\lambda^{(2,1)}_{\dot{\alpha}} &= +4(\xi_m\varepsilon C_{(kl)}\sigma^\mu)_{\dot{\alpha}}\partial_\mu(\bar{\phi})^2u^{-(k}u^{-l}u^{-m)} + O(C^2), \\
\lambda^{(2,2)} &= -i4\sqrt{2}(\xi_m\varepsilon C_{(kl)}\sigma^\mu)_{\dot{\alpha}}\partial_\mu(\bar{\psi}^{\dot{\alpha}}_n\bar{\phi})u^{-(k}u^{-l}u^{-m}u^{-n)} + O(C^2).
\end{aligned} \tag{30}
$$

Then we find the deformed supersymmetry transformation laws in the WZ gauge:

$$
\begin{aligned}
\delta^*_\xi\phi &= -\sqrt{2}i\xi^i\psi_i - \frac{8}{3}i(\xi^j\varepsilon C_{(jk)}\psi^k)\bar{\phi} - \frac{2\sqrt{2}}{3}i(\xi^j\varepsilon C_{(jk)}\sigma^\nu\bar{\psi}^k)A_\nu + O(C^2), \\
\delta^*_\xi\bar{\phi} &= 0, \\
\delta^*_\xi A_\mu &= i\xi^i\sigma_\mu\bar{\psi}_i + 2\sqrt{2}i(\xi^j\varepsilon C_{(jk)}\sigma_\mu\bar{\psi}^k)\bar{\phi} + O(C^2), \\
\delta^*_\xi\psi^{\alpha i} &= -(\xi^i\sigma^{\mu\nu})^\alpha F_{\mu\nu} - D^{ij}\xi^\alpha_j + 2\sqrt{2}D^{(ij}(\xi^{k)}\varepsilon C_{(jk)})^\alpha\bar{\phi} - 2i(\bar{\psi}^{(i}\bar{\psi}^{j)})(\xi^k\varepsilon C_{(jk)})^\alpha \\
&\quad - \left\{2\sqrt{2}(\xi^j\varepsilon C_{(jk)}\sigma^{\mu\nu})^\alpha + \frac{2\sqrt{2}}{3}(\xi^j\sigma^{\mu\nu}\varepsilon C_{(jk)})^\alpha + \sqrt{2}C^{\mu\nu}_{(jk)}\xi^{\alpha j}\right\}\epsilon^{ki}\bar{\phi}F_{\mu\nu} \\
&\quad + \left\{\frac{4\sqrt{2}}{3}(\xi^j\sigma^{\mu\nu}\varepsilon C_{(jk)})^\alpha + 2\sqrt{2}C^{\mu\nu}_{(jk)}\xi^{\alpha j}\right\}\epsilon^{ki}\partial_\mu\bar{\phi}A_\nu \\
&\quad - \frac{2\sqrt{2}}{3}(\xi^j\varepsilon C_{(jk)})^\alpha\epsilon^{ki}\partial^\mu\bar{\phi}A_\mu + O(C^2), \\
\delta^*_\xi\bar{\psi}^i_{\dot{\alpha}} &= +\sqrt{2}(\xi^i\sigma^\nu)_{\dot{\alpha}}\partial_\nu\bar{\phi} + 2(\xi^j\varepsilon C_{(jk)}\sigma^\nu)_{\dot{\alpha}}\partial_\nu(\bar{\phi}^2)\epsilon^{ki} + O(C^2), \\
\delta^*_\xi D^{ij} &= -2i\xi^{(i}\sigma^\nu\partial_\nu\bar{\psi}^{j)} - 6\sqrt{2}i\epsilon^{k(l}\partial_\nu\{(\xi^i\varepsilon C_{(kl)}\sigma^\nu\bar{\psi}^{j)})\bar{\phi}\} + O(C^2).
\end{aligned} \tag{31}
$$

To obtain the expression for $\delta^*_\xi\psi$, we have used the following relation:

$$
(\xi^i\varepsilon C_{(jk)}\sigma^{\mu\nu})^\alpha + (\xi^i\sigma^{\mu\nu}\varepsilon C_{(jk)})^\alpha + \xi^{\alpha i}C^{\mu\nu}_{(jk)} = 0, \tag{32}
$$

which can be proved by explicit calculation. Note that the expression for $\delta_\xi^* \psi$ given above is one of the possible expressions and is chosen so that the invariance of the action can be easily examined.

We can check that the $O(C)$ action (12) is indeed invariant under the deformed supersymmetry transformation (31). Denoting the deformed supersymmetry transformation δ_ξ^* as

$$\delta_\xi^* = \delta_\xi^{*(0)} + \delta_\xi^{*(1)} + \cdots, \tag{33}$$

where $\delta_\xi^{*(n)}$ represents the $O(C^n)$ variations, we can see that

$$\delta_\xi^{*(1)} S_{*,2} + \delta_\xi^{*(0)} S_{*,3} = 0. \tag{34}$$

To show this, we need (32) and a formula $T_{ij} - T_{ji} = \epsilon_{ij}\epsilon^{kl} T_{lk}$ for a given tensor T_{ij}. Note that $\delta_\xi^{*(1)} S_{*,3}$ is non zero for generic deformation parameters. Therefore we need higher order terms in C in order to obtain fully supersymmetric action.

After the redefinition (15), the deformed supersymmetry transformation (31) becomes

$$\begin{aligned}
\delta_\xi^* \hat{\phi} &= -\sqrt{2} i \xi^i \hat{\psi}_i - \frac{8}{3} i (\xi^j \varepsilon C_{(jk)} \hat{\psi}^k) \hat{\bar{\phi}} + O(C^2), \\
\delta_\xi^* \hat{\bar{\phi}} &= 0, \\
\delta_\xi^* \hat{A}_\mu &= i \xi^i \sigma_\mu \hat{\bar{\psi}}_i + 2\sqrt{2} i (\xi^j \varepsilon C_{(jk)} \sigma_\mu \hat{\bar{\psi}}^k) \hat{\bar{\phi}} + O(C^2), \\
\delta_\xi^* \hat{\psi}^{\alpha i} &= -(\xi^i \sigma^{\mu\nu})^\alpha \hat{F}_{\mu\nu} - \hat{D}^{ij} \xi_j^\alpha - i (\xi^i \sigma_{\mu\nu})^\alpha C^{\mu\nu}_{(jk)} (\hat{\bar{\psi}}^j \hat{\bar{\psi}}^k) + 2\sqrt{2} \hat{D}^{(ij} (\xi^{k)} \varepsilon C_{(jk)})^\alpha \hat{\bar{\phi}} \\
&\quad - \left\{ 2\sqrt{2} (\xi^j \varepsilon C_{(jk)} \sigma^{\mu\nu})^\alpha + \frac{2\sqrt{2}}{3} (\xi^j \sigma^{\mu\nu} \varepsilon C_{(jk)})^\alpha + \sqrt{2} C^{\mu\nu}_{(jk)} \xi^{\alpha j} \right\} \epsilon^{ki} \hat{\bar{\phi}} \hat{F}_{\mu\nu} + O(C^2), \\
\delta_\xi^* \hat{\bar{\psi}}^i_{\dot{\alpha}} &= +\sqrt{2} (\xi^i \sigma^\nu)_{\dot{\alpha}} \partial_\nu \hat{\bar{\phi}} + 2 (\xi^j \varepsilon C_{(jk)} \sigma^\nu)_{\dot{\alpha}} \partial_\nu (\hat{\bar{\phi}}^2) \epsilon^{ki} + O(C^2), \\
\delta_\xi^* \hat{D}^{ij} &= -2i \xi^{(i} \sigma^\nu \partial_\nu \hat{\bar{\psi}}^{j)} \\
&\quad - 6\sqrt{2} i \epsilon^{k(l} \partial_\nu \{ (\xi^i \varepsilon C_{(kl)} \sigma^\nu \hat{\bar{\psi}}^{j)}) \hat{\bar{\phi}} \} + 2\sqrt{2} i \epsilon^{il} \epsilon^{jm} (\xi^k \varepsilon C_{(lm)} \sigma^\nu \hat{\bar{\psi}}_k) \partial_\nu \hat{\bar{\phi}} + O(C^2).
\end{aligned} \tag{35}$$

For generic non-singlet deformations, it seems difficult to find the appropriate field redefinition such that both gauge and supersymmetry transformations become canonical.

In this paper we have studied $\mathcal{N} = 2$ supersymmetric $U(1)$ gauge theory in non(anti)-commutative harmonic superspace with the non-singlet deformation parameter C. We have determined deformed $\mathcal{N} = (1, 0)$ supersymmetry transformation at the order C for component fields of the analytic superfield V_{WZ}^{++} in the WZ gauge. We have checked that

the $O(C)$ component action is invariant under this deformed supersymmetry transformation.

It is interesting to study the reduction of deformation parameters such that only $\mathcal{N}=1$ subspace becomes non(anti)commutative. In this case we will be able to construct gauge and $\mathcal{N}=(1,0)$ supersymmetry transformations. The action (7) will reduce to the component action defined in $\mathcal{N}=1/2$ superspace by some field identifications, which is expected to have $\mathcal{N}=(1,1/2)$ supersymmetry[11]. A detailed analysis will appear in a forthcoming paper[24].

Another obvious generalization is the extension to non-abelian gauge groups. For a gauge group $U(N)$, it would be possible to construct the $\mathcal{N}=(1,0)$ supersymmetry in a similar way. In particular, it would be interesting to study the (deformed) central charge in the algebra. Instanton solutions in the deformed gauge theory and its contribution to the prepotential of the low-energy effective theory will be also interesting in viewpoint of its relation to superstring theory with R-R background.

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